Board of Studies in Mathematics (UG)
UNIVERSITY OF KERALA

First Degree Programme in MATHEMATICS
under Choice Based Credit and Semester System

SYLLABUS
for 2018 admission onwards

STRUCTURE of CORE COURSES

Sem	Course Code	Course title	Instr.hrs. per week	Credit
I	MM 1141	Methods of Mathematics	4	4
II	MM 1221	Foundations of Mathematics	4	3
III	MM 1341	Elementary Number Theory and Calculus – I	5	4
IV	MM 1441	Elementary Number Theory and Calculus – II	5	4
V	MM 1541	Real Analysis – I	5	4
	MM 1542	Complex Analysis – I	4	3
	MM 1543	Abstract Algebra – Group Theory	5	4
	MM 1544	Differential Equations	3	3
	MM 1545	Mathematics Software - LaTeX & SageMath (Practical Examination Only)	4	3
	MM 1551	Open Course	3	2
	——	Project preparation - From selecting the topic to presenting the final report	1	
VI	MM 1641	Real Analysis – II	5	4
	MM 1642	Complex Analysis – II	4	3
	MM 1643	Abstract Algebra – Ring Theory	4	3
	MM 1644	Linear Algebra	5	4
	MM 1645	Integral Transforms	4	3
	MM 1651	Elective Course	3	2
	MM 1646	Project		4

STRUCTURE of OPEN COURSES

Sem	Course Code	Course title	Instr.hrs. per week	Credit
V	MM 1551.3	Basic Mathematics	3	2

STRUCTURE of ELECTIVE COURSES

Sem	Course Code	Course title	Instr.hrs. per week	Credit
VI	MM 1661.1	Graph Theory	3	2

Syllabus for the First Degree Programme in Mathematics of the University of Kerala

Semester I

Methods of Mathematics

CODE: MM 1141
Instructional hours per week: 4 No.of credits: 4

In this paper, we quickly review the fundamental methods of solving problems viz. the limiting method, finding the rate of changes through differentiation method, and finding the area under a curve through the integration method.

Module I - Methods of Differential Calculus(36 Hours)

In the beginning of this module, the basic concepts of calculus like limit of functions especially infinite limits and limits at infinity, continuity of functions, basic differentiation, derivatives of standard functions, implicit differentiation etc. should be reviewed with examples.
The above topics which can be found in chapter 2 of text [1] **below are not to be included in the end semester examination.** A maximum of 5 hours should be devoted for the review of the above topics. After this quick review, the main topics to discuss in this module are the following:
Differentiating equations to relate rates, how derivatives can be used to approximate non- linear functions by linear functions, error in local linear approximation, differentials;
Increasing and decreasing functions and their analysis, concavity of functions, points of inflections of a function and applications, finding relative maxima and minima of functions and graphing them, critical points, first and second derivative tests, multiplicity of roots and its geometrical interpretation, rational functions and their asymptotes, tangents and cusps on graphs;
Absolute maximum and minimum, their behaviour on various types of intervals, appli- cations of extrema problems in finite and infinite intervals, and in particular, applications to Economics;
Motion along a line, velocity and speed, acceleration, Position - time curve, Rolle's, Mean Value theorems and their consequences;
Indeterminate forms and L'Hôpital's rule;
The topics to be discussed in this module can be found in chapter 2,3 and 6 of text [1] *below.*

Module II - Methods of Integral Calculus(36 Hours)

The module should begin with revising integration techniques, like integration by sub- stitution, fundamental theorem of calculus, integration by parts, integration by partial fractions, integration by substitition and the concept of definite integrals.

The above topics which can be found in chapter 4 and 7 of text [1] **below are not to be included in the end semester examination.** A maximum of 5 hours should be devoted for the review of the above topics.
After this quick review, the main topics to discuss in this module are the following:
Finding position, velocity, displacement, distance travelled of a particle by integration, analysing the distance-velocity curve, position and velocity when the acceleration is con- stant, analysing the free-fall motion of an object, finding average value of a function and its applications;Area, volume, length related concepts :
Finding area between two curves, finding volumes of some three dimensional solids by various methods like slicing, disks and washers, cylindrical shells, finding length of a plane curve, surface of revolution and its area;
Work done : Work done by a constant force and a variable force, relationship between work and energy;
Relation between density and mass of objects, center of gravity, Pappus theorem and related problems
Fluids, their density and pressure, fluid force on a vertical surface. Introduction

to Hyperbolic functions and their applications in hanging cables;
Improper integrals, their evaluation, applications such as finding arc length and area of surface.
The topics to be discussed in this module can be found in chapter 4, 5, 6 and 7 of text [1] *below.*

Text 1 – H Anton, I Bivens, S Davis. *Calculus*, 10th Edition, John Wiley& Sons

References

Ref. 1 – G B Thomas, R L Finney. *Calculus*, 9th Edition, Addison-Weseley Publishing Company

Ref. 2 – J Stewart. *Calculus with Early Transcendental Functions*, 7th Edition, Cengage India Private Limited

Semester II

Foundations of Mathematics

CODE: MM 1221
Instructional hours per week: 4 No.of credits: 3

The rigorous study of mathematics begins with understanding the concepts of sets and functions. After that, one needs to understand the way in which a mathematician formally makes statements and proves or disproves it. We start this course with an introduction to these fundamental concepts. Apart from that, the basic of vector calculus is to be revised before moving to more advanced topics.

Module I - Foundations of Logic and Proof(36 Hours)

The following are the main topics in this module :
Statements, logical connectives, and truth tables, conditional statements and parts of it, tautology and contradiction, using various quantifiers like universal and existential quantifiers in statements, writing negations, determining truth value of statements;
Proof : Various techniques of proof like inductive reasoning, counter examples, deduc- tive reasoning, hypothesis and conclusion, contrapositive statements, converse statements, contradictions, indirect proofs;
Sets and relations: A review of basic set operations like union, intersection, subset, superset concepts, equality of sets, complements, disjoint sets, indexed family of sets and operations on such families, ordered pairs, relations on sets, cartesian products (finite case only), various types of relations (reflexive, symmetric, transitive, equivalence), partitions of sets;
Functions: domain, codomain, range of functions, one-one, onto, bijective functions, image, preimage of functions, composing functions and the order of composition, inverse functions, cardinality of a set, equinumerous (equipotent) sets

The topics to be discussed in this module can be found in chapter 1 and 2 of text [1] *below.*

Module II - Foundations of co-ordinate geometry(18 Hours)

The following are the main topics in this module :
Parametric equations of a curve, orientation of a curve, expressing ordinary functions parametrically, tangent lines to parametric curves, arc length of parametric curves;
Polar co-ordinate systems, converting between polar and rectangular co-ordinate sys- tems, graphs in the polar co-ordinate system, symmetry tests in the polar co-ordinate system, families of lines, rays, circles, other curves, spirals;
Tangent lines to polar curves, arc length of the curve, area, intersections of polar curves;
Conic sections : definitions and examples, equations at standard positions, sketching them, asymptotes of hyperbolas, translating conics, reflections of conics, applications,rotation of axes and eliminating the cross product term from the equation of a conic, polar equations of conics, sketching them, applications in astronomy such as Kepler's laws, related problems

The topics to be discussed in this module can be found in chapter 10 of text [2] *below.*

Module III - Foundations of vector calculus(18 Hours)

To begin with, the three dimesional rectangular co-ordinate system should be discussed and how distance is to be calculated between points in this sytem. Basic operations on vectors like their addition, cross and dot products should be introduced next. The concept of projections of vectors and the relation with dot product should be given emphasize. Equations of lines determined by a point and vector, vector equations in lines, equatins of planes using vectors normal to be should be discussed. Quadric surfaces which are three dimensinal analogues of conics should be discussed next. Various co-ordinate systems like cylindrical, spherical should be discussed next with the methods for conversion between various co-ordinate systems.

The topics to be discussed in this module can be found in chapter 11 of text [2] *below.*

Texts

Text 1 – S R Lay. *Analysis with an Introduction to Proof*, 5th Edition, Pearson Education Limited

Text 2 – H Anton, I Bivens, S Davis. *Calculus*, 10th Edition, John Wiley & Sons

References

Ref. 1 – J P D'Angelo, D B West. *Mathematical Thinking - Problem Solving and Proofs*, 2nd Edition, Prentice Hall

Ref. 2 – Daniel J Velleman. *How to Prove it : A Structured Approach*, 2nd Edition, Cambridge University Press

Ref. 3 – Elena Nardi, Paola lannonne. *How to Prove it : A brief guide for teaching Proof to Year 1 mathematics undergraduates*, University of East Anglia, Centre for Applied Research in Education

Ref. 4 – G B Thomas, R L Finney. *Calculus*, 9th Edition, Addison-Weseley Publishing Company

Ref. 5 – J Stewart. *Calculus with Early Transcendental Functions*, 7th Edition, Cengage India Private Limited

Semester III

Elementary Number Theory and Calculus – I

CODE: MM 1341
Instructional hours per week: 5 No.of credits: 4

Towards beginning the study on abstract algebraic structures, this course introduces the fundamental facts in elementary number theory. Apart from that, calculus of vector valued functions and multiple integrals is also discussed.

Module I - Divisibility in integers(18 Hours)

The topic of elementary number theory is introduced for further developing the ideas in abstract algebra. The following are the main topics in this module :
The division algorithm, Pigeonhole principle, divisibility relations, inclusion-exclusion principle, base-b representations of natural numbers, prime and composite numbers, infini- tude of primes, GCD, linear combination of integers, pairwise relatively prime integers, the Euclidean algorithm for finding GCD, the fundamental theorem of arithmetic, canonical decomposition of an integer into prime factors, LCM;
Linear Diophantine Equations and existence of solutions, Eulers Method for solving LDE's

The topics to be discussed in this module can be found in chapter 2 and 3 of text [2] *below.*

Module II - Vector valued functions(30 Hours)

Towards going to the calculus of vector valued functions, we define such functions. The other topics in this module are the following :
Parametric curves in the three dimensional space, limits, continuity and derivatives of vector valued functions, geometric interpretation of the derivative, basic rules of differ- entiation of such functions, derivatives of vector products, integrating vector functions, length of an arc of a parametric curve, change of parameter, arc length parametrizations, various types of vectors that can be associated to a curve such as unit vectors, tangent vectors, binormal vectors, definition and various formulae for curvature, the geometrical interpretation of curvature, motion of a particle along a curve and geometrical interpreta- tion of various vectors associated to it, various laws in astronomy like Kepler's laws and problems

The topics to be discussed in this module can be found in chapter 12 of text [1] *below.*

Module III -Multivariable Calculus(42 Hours)

After introducing the concept of functins of more than one variable, the sketching of them in three dimensinal cases with the help of level curves should be discussed. Countours and level surface plotting also should be discussed. The other topics in this module are the following:
Limits and continuity of Multivariable functions, various results related to finding the limits and establishing continuity, continuity at boundary points, partial derivatives of functions, partial derivative as a function, its geometrical interpretation, implicit partial differentiation, changing the order of partial differentiation and the equality conditions;
Differentiability of a multivariate function, differentiability of such a function implies its continuity, local linear approximations, chain rules - various versions, directional derivative and differentiability, gradient and its properties, applications of gradients;
Tangent planes and normal vectors to level surfaces, finding tangent lines to intersec- tions of surfaces, extrema of multivariate functions, techniques to find them, critical and saddle points, Lagrange multipliers to solve extremum problems with constrains,

The topics to be discussed in this module can be found in chapter 13 of text [1] *below.*

Texts

Text 1 – H Anton, I Bivens, S Davis. *Calculus*, 10th Edition, John Wiley & Sons

Text 2 – Thomas Koshy. *Elementary Number Theory with Applications*, 2nd Edition, Academic Press

References

Ref. 1 – G B Thomas, R L Finney. *Calculus*, 9th Edition, Addison-Weseley Publishing Company

Ref. 2 – J Stewart. *Calculus with Early Transcendental Functions*, 7th Edition, Cengage India Private Limited

Ref. 3 – G A Jones, J M Jones. *Elementary Number Theory*, Springer

Semester IV

Elementary Number Theory and Calculus – II

CODE: MM 1441
Instructional hours per week: 5 No.of credits: 4

As in the previous semester, towards beginning the study on abstract algebraic struc- tures, this course introduces the fundamental facts in elementary number theory. Apart from that, calculus of vector valued functions and multiple integrals is also discussed.

Module I - Congruence relations in integers(30 Hours)

Towards defining the congruence classes in Z, we begin with defining the congruence rela- tion. Its various properties should be discussed, and then the result that no prime of the form $4n + 3$ is a sum of two squares should be discussed. The other topics in this module are the following:
Defining congruence classes, complete set of residues, modulus exponentiation, finding re- minder of big numbers using modular arithmetic, cancellation laws in modular arithmetic, linear congruences and existence of solutions, solving Mahavira's puzzle, modular inverses, Pollard Rho factoring method;
Certain tests for divisibility - The numbers here to test are powers of 2, 3, 5, 7, 9, 10, 11, testing whether a given number is a square;
Linear system of congruence equations, Chinese Remainder Theorem and some appli- cations;
Some classical results like Wilson's theorem, Fermat's little theorem, Pollard $p - 1$ factoring method, Eulers' theorem,
The topics to be discussed in this module can be found in chapter 2 and 3 of text [2] *below.*

Module II - Multiple integrals(30 Hours)
Here we discuss double and triple integrals and their applications. The main topics in this module are the follwing:
Double integrals: Defining and evaluating double integrals, its properties, double inte- grals over non rectangular regions, determining limits of integration, revising the order of integration, area and double integral, double integral in polar coordinates and their evaluation, finding areas using polar double integrals, conversion between rectangular to polar integrals, finding surface area, surface of revolution in parametric form, vector valued function in two variables, finding surface area of parametric surfaces;
Triple integrals : Properties, evaluation over ordinary and special regions, determin- ing the limits, volume as triple integral, modifying order of evaluation, triple integral in cylindrical co-ordinates, Converting the integral from one co-ordinate system to other;
Change of variable in integration (single, double, and triple), Jacobians in two vari- ables.
The topics to be discussed in this module can be found in chapter 14 of text [1] *below.*

Module III - Vector Calculus(30 Hours)
After the differentiation of vector valued functions in the last semester, here we introduce the concept of integrating vector valued functions. Some important theorems are also to be discussed here. The main topics are the following :
Vector fields and their graphical representation, various type of vector fields (inverse- square, gradient, conservative), potential functions, divergence, curl, the Q operator, Laplacian;
Integrating a function along a curve (line integrals), integrating a vector field along a curve, defining work done as a line integral, line integrals along piecewise-smooth curves, integration of vector fields and independence of path, fundamental theorem of line inte- grals, line integrals along closed paths, test for conservative vector fields, Green's theorem and applications;

Defining and evaluating surface integrals, their applications, orientation of surfaces, evaluating flux integrals, The divergence theorem, Gauss' Law, Stoke's theorem, applica- tions of these theorems.
The topics to be discussed in this module can be found in chapter 15 of text [1] *below.*

Texts

Text 1 – H Anton, I Bivens, S Davis. *Calculus*, 10th Edition, John Wiley & Sons

Text 2 – Thomas Koshy. *Elementary Number Theory with Applications*, 2nd Edition, Academic Press

References

Ref. 1 – G B Thomas, R L Finney. *Calculus*, 9th Edition, Addison-Weseley Publishing Company

Ref. 2 – J Stewart. *Calculus with Early Transcendental Functions*, 7th Edition, Cengage India Private Limited

Ref. 3 – G A Jones, J M Jones. *Elementary Number Theory*, Springer

Semester V

Real Analysis – I

CODE: MM 1541
Instructional hours per week: 5 No.of credits: 4

In this course, we discuss the notion of real numbers, the ideas of sequence of real numbers and the concept of infinite summation in a formal manner. Many of the topics discussed in the first two modules of this course were introduced somewhat informally in earlier courses, but in this course, the emphasis is on mathematical rigor. A minimal introduction to the metric space structure of R is also included so as to serve as a step- ping stone into the idea of abstract topological spaces. The course is mainly based on Chapters 1–3 of text [1].
All the chapters mentioned above contains a section titled *Discussions* in the beginning of the chapter. This section is intended only for motivating the students, and so should not be made as a part of the examination process.

Module I (25 Hours)
This module introduces the basic concepts about the real number system with some in- troduction to sets, functions, and proof techniques. The following are the main topics to be discussed: existence of an irrational number, the axiom of completeness, upper lower bounds of sets in R, consequences of completeness like Archimedian property of real num- bers, Density of Q in R, existence of square roots, countability of Q and uncountability of R, various cardinality results, Cantor's original proof for uncoubtability of R, and Cantor's theorem on power sets.
The topics to be discussed in this module can be found in chapter 1 of text [1] *below. The first section 1.1 may be briefly discussed and is not meant for examination purposes.*

Module II (40 hours)
Students must have already encountered the idea of infinite series through the example of geometric progression. After discussing the rearrangement concept of infinite series, the following topics are to be introduced rigourously : Limit of a sequence, diverging se- quences, examples, algebraic operations on limits, and order properties of sequences and limits, the Monotone Convergence Theorem, Cauchy's condensation test for convergence of a series, various other tests for the convergence series, the Bolzano-Weierstrass theorem, the Cauchy criterion for convergence of a sequence, rearrangement of absolutely conver- gent series.

The topics to be discussed in this module can be found in chapter 2 of text [1] *below. The first section 2.1 may be briefly discussed and is not meant for examination purposes.*

Module III (25 hours)
This module is intended to be a beginner for learning abstract metric spaces. To motivate the students, the Cantor set should be constructed and shown in the beginning. Then move to the topics open and closed sets in R, and what about their complements, Compactness of sets (defined using sequential convergence), open covers and compactness, perfect and connected sets in R, and finally the Baire's theorem.
The topics to be discussed in this module can be found in chapter 3 of text [1] *below. The first section 3.1 may be briefly discussed and is not meant for examination purposes.*

Texts

Text 1 – Stephen Abbot. *Understanding Analysis*, 2nd Edition, Springer

References

Ref. 1 – R G Bartle, D Sherbert. *Introduction to Real Analysis*, 3rd Edition, John Wiley & Sons

Ref. 2 – W. Rudin. *Principles of Mathematical Analysis*, Second Edition, McGraw-Hill Ref. 3 – Terrence Tao. *Analysis I*, Hindustan Book Agency

Semester V

Complex Analysis – I

CODE: MM 1542
Instructional hours per week: 4 No.of credits: 3
Here we go through the basic complex function theory.

Module I (27 Hours)
Complex numbers : The algebra of Complex Numbers, Point Representation of Complex Numbers, Vectors and Polar forms, The Complex Exponential, Powers and Roots, Planar Sets
Analytic Functions : Functions of a complex variable, Limits and Continuity, Analyticity, The Cauchy Riemann Equations, Harmonic Functions
The topics to be discussed in this module can be found in chapter 1, sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6 and chapter 2, sections 2.1, 2.2, 2.3, 2.4, 2.5 of text [1] *below.*

Module II (15 hours)
Elementary Functions : Polynomials and rational Functions (Proof of the theorem on par- tial fraction decomposition need not be discussed), The Exponential, Trigonometric and Hyperbolic Functions, The Logarithmic Function, Complex Powers and Inverse Trigono- metric Functions.
The topics to be discussed in this module can be found in chapter 3, sections 3.1, 3.2, 3.3, of text [1] *below.*

Module III (30 hours)
Complex Integration : Contours, Contour Integrals, Independence of Path, Cauchy's In- tegral Theorem (Section 4.4a on deformation of Contours Approach is to be discussed, but section 4.4 b on Vector Analysis Approach need not be discussed), Cauchy's Integral Formula and Its Consequences, Bounds of Analytic Functions
The topics to be discussed in this module can be found in chapter 4, sections 4.1, 4.2, 4.3, 4.4a, 4.5 and 4.6 of text [1] *below.*

Texts

Text 1 – Edward B. Saff, Arthur David Snider. *Fundamentals of complex analysis with applications to engineering and science*, 3rd Edition, Pearson Education India

References

Ref. 1 – John H Mathews, Russel W Howell. *Complex Analysis for Mathematics and Engineering*, Jones and Bartlett Publishers

Ref. 2 – Erwin Kreyszig. *Advanced Engineering Mathematics*, 10th Edition, Wiley-India

Ref. 3 – James Brown, Ruel Churchill. *Complex Variables and Applications*, Eighth Edi- tion, McGraw-Hill

Semester V

Abstract Algebra – Group Theory

CODE: MM 1543
Instructional hours per week: 5 No.of credits: 4

The aim of this course is to provide a very strong foundation in the theory of groups. All the concepts appearing in the course are to be supported by numerous examples mainly from the references provided.

Module I (30 Hours)
The concept of group is to be introduced before rigorously defining it. The symmetries of a square can be a starting point for this. After that, definition of group should be stated and should be clarified with the help of examples. After discussing various properties of groups, finite groups and their examples should be discussed. The concept of subgroups with various characterizations also should be discussed. After introducing the definition of cyclic groups, various examples, and important features of cyclic groups and results on order of elements in such groups should be discussed.
The topics to be discussed in this module can be found in chapter 1, 2 3 and 4 of text [1] *below.*

Module II (24 Hours)
This module starts with defining and analysing various properties permutation groups which forms one of the most important class of examples for non abelian, finite groups. After defining operations on permutations, their properties are to be discussed. To moti- vate the students, the example of check-digit scheme should be discussed (This section on check-digit scheme is not meant for the examinations). Then we proceed to define the no- tion of equivalence of groups viz. isomorphisms. Several examples are to be discussed for explaining this notion. The properties of isomorphisms are also to be discussed together with special classes of isomorphisms like automorphisms and inner automorphisms before finishing the module with the classic result of Cayley on finite groups.
The topics to be discussed in this module can be found in chapter 5 and 6 of text [1] *below.*

Module III (18 Hours)
In this module we prove one of the most important results in group theory which is the Langrange's theorem on counting cosets of a finite group. The concept of cosets of a group should be defined giving many examples before proving the Lagrange's theorem. As some of the applications of this theorem, the connection between permutation groups and rotations of cube and soccer ball should be discussed. The section on Rubik's cube and section on internal direct products need not be discussed.
The topics to be discussed in this module can be found in chapter 7 and 9 of text [1] *below.*

Module IV (18 Hours)

Here the concept of group homomorphisms should be defined with sufficient number of examples. After proving the first isomorphism theorem, the fundamental theorem of iso- morphism should be introduced and proved. Classifying groups based on the fundamental theorem should be discussed in detail.
The topics to be discussed in this module can be found in chapter 10 and 11 of text [1] *below.*

Texts

Text 1 – Joseph Gallian. *Contemporary Abstract Algebra*, 8th Edition, Cengage Learning

References

Ref. 1 – D S Dummit, R M Foote. *Abstract Algebra*, 3rd Edition, Wiley Ref. 2 – I N Herstein. *Topics in Algebra*, Vikas Publications

Semester V

Differential Equations

CODE: MM 1544
Instructional hours per week: 3 No.of credits: 3

In this course, we discuss how differential equations arise in various physical problems and consider some methods to solve first order differential equations and second order linear equations. For introducing the concepts, text [1] may be used, and for strengthening the theoretical aspects, reference [1] may be used.

Module I - First order ODE (18 hours)

In this module we discuss first order equations and various methods to solve them. Suffi- cient number of exercises also should be done for understanding the concepts thoroughly. The main topics in this module are the following:
Modelling a problem, basic concept of a differential equation, its solution, initial value problems, geometric meaning (direction fields), seperable ODE, reduction to seperable form, exact ODEs and integrating factors, reducing to exact form, homogeneous and non homogeneous linear ODEs, special equations like Bernoulli equation, orthogonal trajecto- ries, understanding the existence and uniqueness of solutions theorem.

The topics to be discussed in this module can be found in chapter 1 of text [1] *below.*

Module II - Second order ODE (18 hours)

As in the first module, we discuss second order equations and various methods to solve them. Sufficient number of exercises also should be done for understanding the concepts thoroughly. The main topics in this module are the following:
homogeneous linear ODE of second order, initial value problem, basis, and general solu- tions, finding a basis when one solution is known, homogeneous linear ODE with constant coefficients (various cases that arise depending on the characteristic equation), differential operators, Euler-Cauchy Equations, existence and uniqueness of solutions w.r. to wron- skian, solving nonhomogeneous ODE via the method of undetermined coefficients, varius applications of techniques, solution by variation of parameters.

The topics to be discussed in this module can be found in chapter 2 of text [1] *below.*

Texts

Text 1 – Erwin Kreyszig. *Advanced Engineering Mathematics*, 10th Edition, Wiley-India

References

Ref. 1 – G. F. Simmons. *Differential Equations with applications and Historical notes*, Tata McGraw-Hill, 2003

Ref. 2 – H Anton, I Bivens, S Davis. *Calculus*, 10th Edition, John Wiley & Sons

Ref. 3 – Peter V. O' Neil. *Advanced Engineering Mathematics*, Thompson Publications, 2007

Semester V

Mathematics Software – LaTeX & SageMath

CODE: MM 1545

Instructional hours per week: 4 No.of credits: 3

Here we introduce two software which are commonly used by people working in Math- ematics - a science typesetting software LaTeX, and a mathematical computation and visualization software SageMath. The aim of introducing LaTeX software is to enable stu- dents to typeset the project report which is a compulsory requirement for finishing their undergraduate mathematics programme succesfully. The aim of learning SageMath is to enable students to see how the computational techniques they have learned in the previous semesters can be put into action with the help of software so as to reduce human effort. Also, they should be able to use this software for further computations in their own in the forthcoming semester.

Module I - LaTeX for prearing a project report in Mathematics (36 Hours)
Graphical User Interface (GUI)/ Edtitor like Kile or TeXstudio should be used for providing training to the students. The main topics in this module are following:
Typesetting a simple article and compiling it; How spaces are treated in the document;
Document layout : various options to be included in the documentclass command, page styles, splitting files into smaller files, breaking line and page, using boxes (like, mbox) to keep text unbroken across lines, dividing document in to parts like frontmatter, mainmatter, backmatter, chapters, sections, etc, cross referencing with and without page number, adding footnotes;
Emphasizing words with \emph, \texttt, \textsl, \textit, \underline etc.
Basic environments like enumerate, itemize, description, flushleft, flusuright, center, quote, quotation
Controlling enumeration via the enumerate package.
Tables : preparing a table and floating it, the longtable environment;
Typesetting mathematics : basic symbols, equations, operators, the equation en- vironment and reference to it, the displaymath environment, exponents, arrows, basic functions, limits, fractions, spacing in the mathematics environments, matrices, align- ing various objects, multi-equation environments, suppressing numbering for one or more equations, handling long equations, phantoms, using normal text in math mode, control- ling font size, typesetting theorems, defintions, lemmas, etc, making text bold in math mode, inserting symbols and environments (array, pmatrix etc) using the support of GUIs;
Figures : Including JPG, PNG graphics with graphicx package, controlling width, height etc, floating figures, adding captions, the wrapfig package;Adding references/bibliogrpahy and citing them, using the package hyperref to add and control hypertext links, creating presentations with pdfscreen, creating new com- mands;
Fonts : changing font size, various fonts, math fonts,
Spacing : changing line spacing, controlling horizontal, vertical spacing, controlling the margins using the geometry package, fullpage package
Preparing a dummy project with titlepage, acknowledgement, certificates, table of contents (using \tableofcontents), list of tables, table of figures, chapters, sections, bib- liography (using the thebibliography environment). This dummy project should contain atleast one example from the each of the topic in the syllabus, and should be submitted for internal evaluation before the end semester practical examination.

Module II - Doing Mathematics with SageMath(36 hours)
Starting SageMath using a browser, how to use the sage cell server https://sagecell. sagemath.org/, how to use SageMathCloud, creating and saving a sage worksheet, saving
the worksheet to an .sws file, moving it and re-opening it in another computer system; Using sagemath as a calculator, basic functions (square root, logarithm, numeric value,
exponential, trigonometric, conversion between degrees and radians, etc.);
Plotting : simple plots of known functions, controlling range of plots, controlling axes, labels, gridlines, drawing multiple plots on a single picture, *adding* plots, polar plotting, plotting implicit functions, contour plots, level sets, parametric 2D plotting, vector fields plotting, gradients;
Matrix Algebra : Adding, multiplying two matrices, row reduced echelon forms to solve linear system of equations, finding inverses of square matrices, determinants, expo- nentiation of matrices, computing the kernel of a matrix;
Defining own functions and using it, composing functions, multi variate functions; Polynomials : Defining polynomials, operations on them like multiplication and divi-
sion, expanding a product, factorizing a polynomial, finding gcd;
Solving single variable equations, declaring multiple variables, solving multi variable equations, solving system of non linear equations, finding the numerical value of roots of equations;
complex number arithmetic, finding complex roots of equations;
Finding derivatives of functions, higher order derivatives, integrating functions, definite and indefinite integrals, numerical integration, partial fractions and integration,
Combinatorcs & Number theory: Permutations, combinations, finding gcd, lcm, prime factorization, prime counting function, n^{th} prime funcion, divisors of a number, counting divisors, modular arithmetic;
Vector calculus : Defining vectors, operations like sum, dot product, cross product, vector valued functions, divergence, curl, multiple integrals;
Computing Taylor, McClaurins polynomials, minimization and Lagrange multipliers, constrained and unconstrained optimization;

Internal Evaluation : A dummy project report prepared in LaTeX should be submitted as assignment for internal evaluation for 5 marks. Another practical record should be submitted the content of which should be problems and their outputs evaluated using SageMath. This record should be awarded a maximum of 10 marks which is earmarked for the internal evaluation examination.

Problems to be included in the examination:

1. Find all local extrema and inflection points of a function
2. Traffic flow optimization
3. Minimum surface area of packaging
4. Newton's method for finding approximate roots
5. Plotting and finding area between curves using integrals
6. Finding the average of a function
7. Finding volume of solid of revolution
8. Finding solution for a system of linear equations
9. Finding divergence and curl of vector valued functions

10. Using differential calculus to analyze a quintic polynomials features, for finding the optimal graphing window

11. Using Pollard's $p-1$ Method of factoring integers, to try to break the RSA cryptosystem

12. Expressing gcd of two integers as a combination of the integers (Bezout's identity)

References

Ref. 1 – Tobias Oetiker, Hubert Partl, Irene Hyna and Elisabeth Schlegl. *The (Not So) Short Introduction to LATEX2e*, Samurai Media Limited (or available online at http://mirrors.ctan.org/info/lshort/english/lshort.pdf)

Ref. 2 - Leslie Lamport. *LATEX: A Document Preparation System*, Addison-Wesley, Reading, Massachusetts, second edition, 1994

Ref. 3 - *LATEX Tutorials—A Primer*, Indian TeX Users Group, available online at https://www.tug.org/twg/mactex/tutorials/ltxprimer-1.0.pdf

Ref. 4 - H. J. Greenberg. *A Simplified introduction to LATEX*, available online at https://www.ctan.org/tex-archive/info/simplified-latex/

Ref. 5 – *Using Kile - KDE Documentation*, https://docs.kde.org/trunk4/en/extragear-office/kile/quick_using.html

Ref. 6 – *TeXstudio : user manual*, http://texstudio.sourceforge.net/manual/current/ usermanual_en.html

Ref. 7 – *The longtable package - TeXdoc.net*, http://texdoc.net/texmf-dist/doc/latex/ tools/longtable.pdf

Ref. 8 – *wrapfig - TeXdoc.net*, http://texdoc.net/texmf-dist/doc/latex/wrapfig/ wrapfig-doc.pdf

Ref. 9 – *The geometry package*, http://texdoc.net/texmf-dist/doc/latex/geometry/ geometry.pdf

Ref. 10 – *The fullpage package*, http://texdoc.net/texmf-dist/doc/latex/preprint/ fullpage.pdf

Ref. 11 – *The SageMathCloud*, https://cloud.sagemath.com/

Ref. 12 – Gregory V. Bard. *Sage for Undergraduates,* American Mathematical Society, available online at http://www.gregorybard.com/Sage.html

Ref. 13 – Tuan A. Le and Hieu D. Nguyen. *SageMath Advice For Calculus* available online at http://users.rowan.edu/~nguyen/sage/SageMathAdviceforCalculus.pdf

Semester V

Project preparation - From selecting the topic to presenting the final report

Instructional hours per week: 1

To complete the undergraduate programme, the students should undertake a project and prepare and submit a project report on a topic of their choice in the subject math- ematics or allied subjects. The work on the project should start in the beginning of the 5th semester itself, and should end towards the middle of the 6th semester. This course (without any examination in the 5th semester, with a project report submissin and project viva in the 6th semester) is introduced for making the students understand various con- cepts behind undertaking such a project and preparing the final report. Towards the end of this course the students should be able to choose and prepare topics in their own and they should understand the layout of a project report.
To quickly get into the business, the first chapter of text [1] may be completely dis- cussed. Apart from that, for detailed information, the other chapters in this book may be used in association with the other references given below. The main topics to discuss in this course are the following:
Quick overview : The structure of Dissertation, creating a plan for the Dissertation, planning the results section, planning the introduction, planning and writing the abstract, composing the title, figures, tables, and appendices, references, making good presentations, handling resources like notebooks, library, computers etc., preparing an interim report. **Topics in detail :** Planning and Writing the Introduction, Planning and Writing the Results, Figures and Tables, Planning and Writing the Discussion, Planning and Writing the References, Deciding On a Title and Planning and Writing the Other Bits, Proofread- ing, Printing, Binding and Submission, oral examinations, preparing for viva, Taking the Dissertation to the Viva
Layout : Fonts and Line Spacing, Margins, Headers, and Footers, Alignment of Text, Titles and Headings, Separating Sections and Chapters

Texts

Text 1 – Daniel Holtom, Elizabeth Fisher. *Enjoy Writing Your Science Thesis or Dis- sertation – A step by step guide to planning and writing dissertations and theses for undergraduate and graduate science students*, Imperial College Press

References

Ref. 1 – Kathleen McMillan, Jonathan Weyers. *How to write Dissertations & Project Reports*, Pearson Education Limited

Ref. 2 – Peg Boyle Single. *Demystifying dissertation writing : a streamlined process from choice of topic to final text*, Stylus Publishing, Virginia

Semester VI

Real Analysis – II

CODE: MM 1641

Instructional hours per week: 5 No.of credits: 4

In the second part of the Real Analysis course, we focus on functions on R, their continuity, existence of derivatives, and integrability. The course is mainly based on Chapters 4,5 and 7 of text[1].
All the chapters mentioned above contains a section titled *Discussions* in the begin- ning of the chapter. These sections are intended only for motivating the students, and so should not be made a part of the examination process.

Module I (35 Hours)
Here we move towards the basic notion of limits of functions and their continuity. Various version of definition of limits are to be discussed here. The algebra of limits of functions and the divergence criterion for functional limits are to be discussed next. The other topics to be discussed in this module are the discontinuity criterion, composition of functions and continuity, continuity and compct sets, results on uniform continuity, the intermediate value theorem, Monotone functions and their continuity.
The topics to be discussed in this module can be found in chapter 4 of text [1] *below. The first section 4.1 may be briefly discussed and is not meant for examination purposes.*

Module II (25 hours)
Here we discuss the derivative concept more rigorously than what was done in the previ- ous calculus courses. After (re)introducing the definition of differentiability of functions, we verify that differentiability implies continuity. Algebra and composing of differen- tiable functions should be discussed next. The interior extremum theorem and Darboux's theorem should be discussed after that. The mean value theorems should be discussed and proved, and the module ends with L'Hospitals results. A continuous everywhere but notwhere differentiable function should be discussed, but it is not meant for the examina- tion. It may be infact used for student seminars.
The topics to be discussed in this module can be found in chapter 5 of text [1] *below. The sections 5.1 and 5.4 may be briefly discussed and is not meant for examination purposes.*

Module III (30 hours)
In the last module, the theory of Riemann integration is to be discussed. Main topics to be included in this module are defining the Riemann integral using upper, lower Riemann sums, and the integrability criterion, continuity and the existence of integral, algebraic operations on integrable functions, (The results and examples on convergence of sequence of functions and integrability may be omitted), the fundamental theorem of calculus and its proof, Lebesgue's criterion for Riemann integrability.

The topics to be discussed in this module can be found in chapter 7 of text [1] *below. The first section 7.1 may be briefly discussed and is not meant for examination purposes.*

Texts

Text 1 – Stephen Abbot; *Understanding Analysis*, 2nd Edition, Springer

References

Ref. 1 – R G Bartle, D Sherbert ; *Introduction to real analysis*, 3rd Edition, John Wiley &

Sons

Ref. 2 – W. Rudin, *Principles of Mathematical Analysis*, Second Edition, McGraw-Hill Ref. 3 – Terrence Tao; *Analysis I*, Hindustan Book Agency

Semester VI

Complex Analysis – II

CODE: MM 1642
Instructional hours per week: 4 No.of credits: 3

Module I (32 Hours)
Series Representations for Analytic Functions : Sequences and Series, Taylor Series, Power Series, Mathematical Theory of Convergence, Laurent series, Zeros and Singularities, The point at Infinity. *The topics to be discussed in this module can be found in chapter 5, sections 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7 of* text [1] *below*.

Module II (20 Hours)
Residue Theory : The Residue Theorem, Trigonometric Integrals over $[0, 2\pi]$, Improper integrals of Certain functions over $[-\infty, \infty]$, Improper integrals involving Trigonometric Functions, Indented Contours
The topics to be discussed in this module can be found in chapter 6, sections 6.1, 6.2, 6.3, 6.4, 6.5 of text [1] *below*.

Module III (20 Hours)
Conformal Mapping : Geometric Considerations, Mobius Transformations
The topics to be discussed in this module can be found in chapter 7, sections 7.2, 7.3, 7.4 of text [1] *below*.

Texts

Text 1 – Edward B. Saff, Arthur David Snider. *Fundamentals of complex analysis with applications to engineering and science*, 3rd Edition, Pearson Education India

References

Ref. 1 – John H Mathews, Russel W Howell. *Complex Analysis for Mathematics and Engineering*, 6th Edition, Jones and Bartlett Publishers

Ref. 2 – Murray R Spiegel. *Complex variables: with an introduction to conformal mapping and its applications*, Schaum's outline.

Ref. 3 – Erwin Kreyszig. *Advanced Engineering Mathematics*, 10th Edition, Wiley-India Ref.

4 – James Brown, Ruel Churchill. *Complex Variables and Applications*, Eighth Edition,

McGraw-Hill

Semester VI

Abstract Algebra – Ring Theory

CODE: MM 1643
Instructional hours per week: 4 No.of credits: 3

After discussing the theory of groups thoroughly in the previous semester, we move towards the next higher algebraic structure rings. As in the last semester, all the new concepts appearing in the course is to be supported by numerous examples mainly from the references provided.

Module I (24 Hours)
The concept of rings, subrings with many examples should be discussed here. Next comes the definition and properties of integral domains, fields, and the characteristic of rings. Ideals, how factor rings are defined using ideals, should be explained next. The definition of prime and maximal ideals with examples should be discused after that.
The topics to be discussed in this module can be found in chapter 12, 13 and 14 of text [1] *below.*

Module II (24 Hours)
After introducing the definition of ring homomorphisms, their properties should be dis- cussed. The field of quotients of an integral domain should be discussed next. The next topic is the definition and various properties of polynomial rings over a commutative ring. Various results on operations on polynomials such as division algorithm, factor theorem, remainder theorem etc should be discussed next. The definition and examples of PID's should be discussed next, before moving to the factorization of polynomials. Tests of irre- ducibility and reducibility and the unique factorization of polynomials over special rings should be discused. .
The topics to be discussed in this module can be found in chapter 15, 16 and 17 of text [1] *below.*

Module III (24 Hours)
In the last module, we introduce more rigorous topics like various type of integral domains. The divisibility properties of integral domains and definition of primes in a general ring should be introduced. Unique factorization domains and the Euclidean domains should be discussed next with examples. Results on these special integral domains are aslo to be discussed.
The topics to be discussed in this module can be found in chapter 18 of text [1] *below.*

Texts

Text 1 – Joseph Gallian; *Contemporary Abstract Algebra*, 8th Edition, Cengage Learning

References
Ref. 1 – D S Dummit, R M Foote; *Abstract Algebra*, 3rd Edition, Wiley Ref. 2 – I N Herstein, *Topics in Algebra*, Vikas Publications

Semester VI

Linear Algebra

CODE: MM 1644

Instructional hours per week: 5 No.of credits: 4

The main focus of this course is to introduce linear algebra and methods in it for solving practical problems.

Module I (15 Hours)
This module deals with a study on linear equations and their geometry. After introducing the geometrical intrepretation of linear equations, following topics should be discussed: various operations on column vectors, technique of Gaussian elimination, operations in- volving elementary matrices, interchanging of rows using elementary matrices, triangular factorisation of matrices and finding inverse of matrices by the elimination method.
The topics to be discussed in this module can be found in chapter 1 of text [1] *below. The section 1.7 may be omitted.*

Module II (25 hours)
Towards the study of vector spaces, specifically R^n, we define them with many examples. Subspaces are to be defined next. After discussing the idea of nullspace of a matrix. The solving linear equations (which was one to some extent in the first module) and finding solutions to non-homogeneous systems from the corresponding homogeneous systems. Af- ter this, linear independence and dependence of vectors, their spanning, basis for a space, its dimension concepts are to be introduced. The column, row, null, left null spaces of a matrix is to be discussed next. When inverses of a matrix exists related to its column/row rank should be discussed. Towards the end of this module, linear transformations (through matrices) and their properties are to be discussed. Types of transformations like rotations, projections, reflections are to be considered next.
The topics to be discussed in this module can be found in chapter 2 of text [1] *below. The section 2.7 on graphs and networks may be omitted.*

Module III (25 hours)
This module is intended for making the idea and concepts of determinants stronger. Its properties like what happens when rows are interchanged, linearlity of expansion along the first row, etc are to be discussed. Breaking a matrix into triangular, diagonal forms and finding the determinants, expansion in cofactors, their applications like solving system of equations, finding volume etc are to be discussed next.
The topics to be discussed in this module can be found in chapter 4 of text [1] *below.*

Module IV (25 hours)
Here we conclude our analysis of matrices. The problem of finding eigen values a matrix is to be introduced first. Next goal is to diagonalize a matrix. This concept should be discussed first, and move to the discussion on the use of eigen vectors in diagonalization. Applications of finding the powers of matrices should be discussed next. The applications like the concept of Markov Matrices, Positive Matrices and their applications in Economics should be discussed. Complex matrices and operations on them are to be introduced next. The concept orthoganality of vectors may be required here from one of the previous sections in text [1] and it should be briefly introduced and discussed here. The module ends with similar matrices, and similarity transformation related ideas. How to diagonalize some special matrices like symmetric and Hermitial matrices are also to be discussed in this module.

The topics to be discussed in this module can be found in chapter 5 of text [1] *below. The section 5.4 on applications to differential equations may be omitted*

Texts

Text 1 – Gilbert Strang, *Linear Algebra and Its Applications*, 4th Edition, Cengage Learning

References

Ref. 1 – *Video lectures of Gilber Strang Hosted by MITOpenCourseware* available at https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/ video-lectures/

Ref. 2 – Thomas Banchoff, John Wermer; *Linear Algebra Through Geometry*, 2nd Edition, Springer

Ref. 3 – T S Blyth, E F Robertson: *Linear Algebra*, Springer, Second Edition. Ref. 4 –

David C Lay: *Linear Algebra*, Pearson

Ref. 5 – K Hoffman and R Kunze: *Linear Algebra*, PHI

Semester VI

Integral Transforms

CODE: MM 1645
Instructional hours per week: 4 No.of credits: 3

After completing courses in ordinary differential equations and basic integral calculus, we see here some of its applications.

Module I(38 Hours)
Laplace Transforms : Laplace Transform. Linearity. First Shifting Theorem (s-Shifting), $s-$ Shifting: Replacing s by $s - a$ in the Transform, Existence and Uniqueness of Laplace Transforms, Transforms of Derivatives and Integrals. ODEs, Laplace Transform of the Integral of a Function, Differential Equations, Initial Value Problems, Unit Step Function (Heaviside Function), Second Shifting Theorem ($t-$Shifting) Time Shifting ($t-$Shifting): Replacing t by $t - a$ in $f(t)$, Short Impulses. Diracs Delta Function. Partial Fractions Convolution , Application to Nonhomogeneous Linear ODEs, Differentiation and Integra- tion of Transforms, ODEs with Variable Coefficients, Integration of Transforms, Special Linear ODEs with Variable Coefficients, Systems of ODEs
The topics to be discussed in this module can be found in sections 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7 of text [1] below.

Module II(34 hours)
Fourier Series, Basic Examples, Derivation of the Euler Formulas, Convergence and Sum of a Fourier Series, Arbitrary Period. Even and Odd Functions. Half-Range Expansions From Period 2π to any Period $P = 2L$, Simplifications: Even and Odd Functions, Half- Range Expansions, Fourier Integral, From Fourier Series to Fourier Integral, Applications of Fourier Integrals, Fourier Cosine Integral and Fourier Sine Integral, Fourier Cosine and Sine Transforms, Linearity, Transforms of Derivatives, Fourier Transform, Complex Form of the Fourier Integral, Fourier Transform and Its Inverse, Linearity. Fourier Transform of Derivatives, Convolution.
The topics to be discussed in this module can be found in Sections 11.1, 11.2, 11.7, 11.8, 11.9 (Excluding Physical Interpretation: Spectrum and Discrete Fourier Transform (DFT),Fast Fourier Transform (FFT)) of text [1] below.

Texts

Text 1 – Erwin Kreyszig. *Advanced Engineering Mathematics*, 10th Edition, Wiley-India

References

Ref. 1 – Peter V. O' Neil, *Advanced Engineering Mathematics*, Thompson Publications, 2007

Ref. 2 – M Greenberg, *Advanced Engineering Mathematics*, 2nd Edition, Prentice Hall

Semester VI

Graph Theory (Elective)

CODE: MM 1661.1
Instructional hours per week: 3 No. of credits: 2

Overview of the Course: The course has been designed to build an awareness of some of the fundamental concepts in Graph Theory and to develop better understanding of the subject so as to use these ideas skillfully in solving real world problems.

Module I (27 Hours)
Basics : The Definition of a Graph, Graphs as Mathematical Models, other basic concepts and definitions, Vertex Degrees, Subgraphs, Paths and Cycles, The Matrix Representation of Graphs, Fusing graphs (The fusion algorithm for connectedness need not be discussed). Trees and Connectivity : Definitions and Simple Properties of trees, Bridges, Spanning Trees, Cut Vertices and Connectivity *The topics in this module can be found in Chapter 1, Sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7 and 1.8, Chapter 2, Sections 2.1, 2.2, 2.3 and of* text [1].

Module II(27 Hours)
Euler Tours and Hamiltonian Cycles : Euler Tours (Fleury's algorithm need not be dis- cussed), The Chinese Postman Problem (Only Statement of the problem is to be discussed)
, Hamiltonian Graphs, The Travelling Salesman Problem (Only Statement of the problem is to be discussed, The Two-Optimal Algorithm and The Closest Insertion Algorithm need not be discussed)
Planar Graphs : Plane and Planar Graphs, Euler's Formula, The Platonic Bodies, Kuratowski's Theorem (Without proof).
The topics in this module can be found in Chapter 3, Sections 3.1, 3.2, 3.3 and 3.4, Chapter 5, Sections 5.1, 5.2, 5.3 and 5.4 of text [1].

Texts

Text 1 – John Clark, Derek Allan Holton. *A first look at Graph Theory*, World Scientific

References

Ref. 1 – R Balakrishnan, Ranganatahan. *A Text Book of Graph Theory*, 2nd Edition, Springer

Ref. 2 – V Balakrishnan. *Graph Theory*, Schaums Outline

Ref. 3 – J A Body, U S R Murthy. *Graph Theory with Applications*, The Macmillan Press